Physics problems at a tau–charm factory

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In connection with the project of a tau–charm factory (τcF) at Dubna, the possible physics problems that could be solved with this accelerator are briefly reviewed. It is emphasized that experiments at a τcF will make it possible to study the τ lepton and the c quark with an accuracy comparable to the accuracy with which the lighter leptons and quarks have been investigated.

At the Joint Institute for Nuclear Research at Dubna a project of a new e^+e^- collider, a so-called tau–charm factory (τcF), is being developed. The new accelerator is intended to have the following parameters:

Total energy, GeV	3–5
Luminosity, 10^{33} cm$^{-2}\cdot$sec^{-1}	1(1.5)
Length of perimeter, m	300
Energy spread in beam at energy 1.5 GeV, keV	50
Number of crossings	2

We discuss here the possible program of physics investigations at a tau–charm factory.[1] We begin with a very brief review of the present status of the standard model of the electroweak interaction. The seventies and eighties were truly a golden time in elementary-particle physics. We mention the most famous discoveries: neutral currents (1973), ψ particles (1974), the tauon (1975), charmed particles (1975), the Υ particles (1977), the B particles (eighties), intermediate W and Z bosons (1983), and only three neutrino species (1989–1990). The detailed quantitative study of the various processes due to charged and neutral currents undoubtedly indicates that the Glashow–Weinberg–Salam model makes it possible to describe a wide range of electroweak processes in the entire accessible range of energies.

Data obtained at LEP to measure the number of neutrino species[2] indicate that there are three generations of leptons and quarks:

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}\begin{pmatrix}u\\ d\end{pmatrix},\ \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}\begin{pmatrix}c\\ s\end{pmatrix},\ \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}\begin{pmatrix}t\\ b\end{pmatrix}. \quad (1)$$

However, we emphasize that not all the particles in these three generations have been discovered: 1) there are no direct proofs for the existence of ν_τ; 2) as yet, the t quark has not been discovered. It follows from LEP data on measurement of the Z-boson mass, and also from the CDF-UA2 data on the measurement of the ratio m_W/m_Z and from data on measurement of the parameter $\sin^2\theta_W$ in deep inelastic scattering of neutrinos by nucleons that $m_t = 140 \pm 45$ GeV (Ref. 3). It follows from the CDF data that $m_t > 89$ GeV (Ref. 4).

Among other problems of the standard model, we mention the following:

A. The problem of the Higgs boson. It has not yet been discovered. It follows from LEP data that the mass of the standard Higgs boson is greater than 40 GeV.

B. The problem of CP invariance. We do not yet know whether the standard model can explain quantitatively the phenomenon of CP invariance and whether there exist other sources of CP violation.

C. The problem of the neutrino masses and mixing. We do not yet know whether the neutrino masses are nonvanishing and whether the current field of the neutrinos ν_{lL} is a superposition of the left components of the neutrino fields with definite masses. The problem of conservation of the leptonic charges is intimately related to the neutrino mass problem. Are the conservation laws for the leptonic charges exact or only approximate? At the present time, numerous experiments are being made with the aim of answering these questions. The corresponding experiments at a tau–charm factory could be very important.

As is well known, the standard model of the electroweak interaction is a far from complete theory and requires generalization. The following points should be made about this model:

1. There are more than 20 arbitrary fundamental parameters (masses, coupling constants, mixing angles).

2. There is not even a hint of a possibility of solving the generation problem (the standard model is simply based on the assumption that there exist in nature several generations of leptons and quarks, which differ only in the masses of the particles).

3. There exists the so-called scale hierarchy problem (if it is assumed that the standard model is valid up to the grand-unification scale, then the radiative correction to the mass of the Higgs boson is many orders of magnitude greater than its mass determined by the scale of the breaking of the original symmetry of the standard theory; a supersymmetric generalization of the standard model is one of the possible ways of solving the hierarchy problem).

4. Finally, the standard model unifies only the weak and electromagnetic interactions. It is not a unified theory of all the known interactions.

These and other problems stimulate searches for the most varied ways to go beyond the standard model. It is clear that experiments which look for effects that cannot be described by the standard model are exceptionally important. There exist two ways that can lead to the discovery of such effects at accelerators:

1. The energy of the accelerators can be increased.

2. The accuracy of the measurements can be improved.

Factories make it possible to obtain record accuracies

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of measurements. At the present time, essentially only the Z-boson factory (LEP, CERN) exists. At Novosibirsk and Frascati, φ factories are being constructed (c.m.s. energy 0.3–1.5 GeV). Possibilities for constructing tau–charm factories are being considered in Spain, at Dubna (JINR), and in Moscow (ITEP). At Novosibirsk, Stanford, and Cornell, the possibilities for constructing B factories are being discussed.

One of the important tasks of e^+e^- factories is the detailed study of the fundamental fermions that are the members of the three families. It should be emphasized here that only

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

and, to a lesser degree, $\binom{-}{s}$ have been studied in detail; $\binom{-}{\tau}\binom{c}{b}$ have been studied very poorly, and $\binom{\nu_t}{-}\binom{t}{-}$ have not yet been discovered.

The accelerators whose possibilities we shall discuss here will make it possible to study in detail $\binom{-}{\tau}\binom{c}{-}$, and also to obtain some information about ν_τ. The possibility of building such an accelerator was first considered in Novosibirsk in 1978, and then at CERN during 1987–1988. [5] At the present time, projects for tau–charm factories are being developed at Dubna,[6] in Moscow,[7] and in Spain.[8]

We now turn to a brief discussion of the physics problems that can be solved at tau–charm factories.[1] We begin with the physics of the τ leptons. Tauons are produced in the process

$$e^+ + e^- \to \tau^+ + \tau^-. \tag{2}$$

It is expected that at the threshold energy ($E = 3.57$ GeV) the number of $\tau^+\tau^-$ pairs produced per year at a τcF will be 4×10^6, while at energy 4.25 GeV [the maximum of the cross section of the process (2)] the number will be 4×10^7 pairs per year. Thus, the number of $\tau^+\tau^-$ pairs that will be produced per year at a τcF is much greater than the total number of $\tau^+\tau^-$ pairs produced at the existing accelerators ($\cong 10^5$ $\tau^+\tau^-$ paris).

The following features will be characteristic of experiments to study the physics of τ leptons at a τcF: 1) copious statistics; 2) the possibility of measuring the background (by making measurements at energies below the $\tau^+\tau^-$ pair production threshold); 3) good possibilities for identifying the particles; 4) possibilities for tagging particles (for example, tagging of τ^- by detection of the decay $\tau^+ \to \pi^+\nu_\tau$).

One of the important tasks that can be solved at a τcF is that of measuring the tauon mass with accuracy appreciably higher than is currently achieved. For the tauon mass[9]

$$m_\tau = 1784.1^{+2.7}_{-3.6}\ \text{MeV}.$$

For comparison, we give the value of the muon mass:[9]

$$m_\mu = (105.658387 \pm 0.000034)\ \text{MeV}.$$

At a τcF, the mass of the τ lepton will be measured[10] with an error not greater than 0.1 MeV [by studying the energy behavior of the cross section of the process (2) and also by scanning with respect to the energy near the threshold of the reaction (2)].

An accurate value of the tauon mass is needed to obtain information about the mass of the tauon neutrino. At the present time,[11]

$$m_{\nu_\tau} < 35\ \text{MeV}.$$

We note that for the masses of the muon and electron neutrinos we know[12]

$$m_{\nu_\mu} < 0.27\ \text{MeV}; \quad m_{\nu_e} < 9.4\ \text{eV}.$$

Various decay channels of the τ lepton have been considered with the aim of finding the channel most sensitive to the ν_τ mass.[13] It has been shown that the most accurate information about the ν_τ mass can be obtained by studying the decay $\tau \to \nu 5\pi$. At a τcF, a year of work will be sufficient to reduce the upper limit of the ν_τ mass to 3 MeV (possibly 1 MeV).[10,13]

We now turn to the leptonic decays

$$\tau \to e\nu_\tau\nu_e, \quad \tau \to \mu\nu_\mu\nu_\tau. \tag{3}$$

In the case of polarized τ leptons, the probabilities of the decays (3) are determined by four parameters: ρ and η (which characterize the spectrum) and ξ and δ (which characterize the asymmetry). It follows from the standard model that $\rho = 0.75$, $\eta = 0$, $\xi = 1$, $\delta = 0.75$. In the case of the τ leptons, only the Michel parameter ρ has been measured:[14]

$$\rho = 0.747 \pm 0.045.$$

For comparison, we give the values of the parameters ρ..., measured in μ decay:[14]

$$\rho = 0.7518 \pm 0.0026; \quad \eta = -0.007 \pm 0.013;$$

$$\delta = 0.755 \pm 0.009; \quad \xi = 1.0027 \pm 0.0079.$$

After a year of work at a τcF it will be possible to measure the parameters ρ... in leptonic decays of the tauon with an error comparable to the error with which these parameters are measured in μ decay:[15]

$$\rho(+0.003),\ \eta(+0.01),\ \delta(+0.01),\ \xi(+0.01).$$

The widths of the decays

$$\tau \to \nu_\tau\pi, \quad \tau \to \nu_\tau K \tag{4}$$

are determined by the constants f_π and f_K and, therefore, can be predicted. Table I gives the ratios $R_\pi = \Gamma(\tau \to \nu_\tau\pi)/\Gamma(\tau \to \nu_\tau e\nu_e)$ and $R_K = \Gamma(\tau \to \nu_\tau K)/\Gamma(\tau \to \nu_\tau e\nu_e)$ (the second column gives the calculated ratios with allowance for radiative corrections, and the third gives the experimental data). At a τcF, the ratios R_π and R_K will be measured with accuracy 0.5–1%. Thus, the e–μ–τ universality of the charged weak current will be tested at a τcF at the level of the radiative corrections.

The branching ratio of the decay

$$\tau \to \nu_\tau\rho \tag{5}$$

TABLE I.

Ratio	Calculation (with radiative corrections)	Experiment
$R\pi$	0,601	0,62 + 0,04
RK	0,0399	0,038 + 0,011

is 22.8 ± 1.0%. Using the hypothesis of a conserved vector current, it is easy to show that the matrix element of this process is related to the matrix element of the process $e^+e^-\to\rho\to\pi\pi$. Thus, measurement of the width of the decay (5) at a τcF will make possible a high-precision test of CVC at $q^2 = m_\rho^2$.

The decay

$$\tau\to\nu_\tau\pi\eta \qquad (6)$$

takes place because of violation of G parity. This decay is also allowed if the Hamiltonian contains not only the standard first-class current but also a charged second-class current. If allowance is made for the breaking of isotopic invariance (because $m_u\neq m_d$), then for the branching ratio (6) we can obtain the estimate[16]

$$Br(\tau\to\nu_\tau\pi\eta)\cong 10^{-4}-10^{-6}. \qquad (7)$$

It follows from the data currently available that

$$Br(\tau\to\nu_\tau\pi\eta)<0.2\cdot 10^{-2}.$$

It is expected that the limit (7) can be reached at a τcF.

Significant progress could be achieved at a τcF in the search for processes in which the conservation law of the lepton numbers is violated. Table II gives upper limits for the branching ratios of some of these processes. We recall that in the case of the muon $Br(\mu\to e\gamma)<5\times 10^{-11}$ and $Br(\mu\to 3e)<1\times 10^{-12}$. After a year of work at a τcF it will be possible to achieve the level 4×10^{-8} in the study of processes in which the conservation law for the lepton numbers is violated.[17]

The branching ratio of the leptonic decays

$$D_s\to l_{\nu_l} \qquad (8)$$

is given by

$$Br(D_s\to l\nu_l)=\tau_{D_s}\frac{G^2}{8\pi}f_{D_s}^2 m_{D_s} m_l^2 |V_{cs}|^2(1-m_l^2/m_{D_s}^2).$$

Here, G is the Fermi constant; f_{D_s} is the decay constant; V_{cs} is the Kobayashi–Maskawa matrix element. The decays (8) have not hitherto been observed. At a τcF the widths of the decays $D_s\to\mu\nu_\mu$ and $D_s\to\tau\nu_\tau$ will be measured after a year with an accuracy of 1–2%.[10] Thus, the constant f_{D_s} will be measured at a τcF with error $\cong 1\%$. Measurement of the ratio $\Gamma(D_{D_s}\to\mu\nu)/\Gamma(D_s\to\tau\nu_\tau)$ will make it possible to verify μ–τ universality of the weak interaction with error 1–2%, which is comparable with the error in the verification of μ–e universality [based on measurement of the ratio $\Gamma(\pi\to e\nu_e)/\Gamma(\pi\to\mu\nu_\mu)$]. We note also that detailed study of the Cabibbo-allowed decays $D\to Ke\nu_e$ and the Cabibbo-forbidden decays $D\to\pi e\nu_e$ will make it possible to determine the moduli $|V_{cs}|$ and $|V_{cd}|$ of the Kobayashi–Maskawa matrix elements with an error at the percent level (at the present time, these quantities are known with error $\cong 20\%$).

One of the important problems that can be solved at a τcF is that of measuring the D^0–$\bar{D}^0$ mixing parameter. It is well known that the B^0–$\bar{B}^0$ mixing parameter is

$$r_B=\frac{P(B^0\to\bar{B}^0)}{P(B^0\to B^0)}=0.182\pm 0.08,$$

where $P(B^0\to\bar{B}^0)$ is the averaged probability of the decay $B^0\to\bar{B}^0$. The B^0–$\bar{B}^0$ mixing parameter was found to be large because B^0 and $\bar{B}^0$ consist of "lower" quarks and are transformed into each other through "upper" quarks (we have in mind box-type diagrams). The main contribution to r_B is made by the heavy t quark with $m_t>90$ GeV (the parameter r_B is proportional to m_t^2). On the other hand, D^0 and

TABLE II.

Process	Upper bound
$\tau\to\mu\gamma$	$5,2\cdot 10^{-4}$
$\tau\to e\gamma$	$2,0\cdot 10^{-4}$
$\tau\to 3\mu$	$2,9\cdot 10^{-5}$
$\tau\to 3e$	$3,8\cdot 10^{-5}$
$\tau\to e\pi$	$1,4\cdot 10^{-4}$
$\tau\to e\rho$	$3,9\cdot 10^{-5}$

$\overline{D}^0$ are bound states of the upper quarks and are transformed into each other through the lower quarks, the heaviest of which is the b quark with mass $m_b = 5$ GeV. As a result, the following estimate can be obtained[18] from the standard model for the parameter r:

$$r_D = 10^{-4} - 10^{-5}.$$

It follows from existing data that

$$r_D < 5\cdot 10^{-3}.$$

At a τcF, by observing

$$e^+ + e^- \to \psi(3770) \to D^0 + \overline{D}^0 \quad \llcorner\!\!\to K^-\pi^+ \to K^-\pi^+,$$

$$e^+ + e^- \to \psi(3770) \to D^0 + \overline{D}^0 \quad \llcorner\!\!\to K^- e^+ \nu_e \to K^- e^+ \nu_e$$

and other processes, it will be possible[19] to measure the parameter r_D (if its value is as predicted by the standard model).

We now turn to a brief discussion of charmonium physics at a τcF. It is expected that at a "standard" τcF the number of J/ψ particles produced in a year will be 10^{10} (at the present time, about 10^7 decays of J/ψ have been investigated). The $^1P_1(h_{c1})$ charmonium state has not hitherto been observed in e^+e^- experiments (some indications of the existence of this state were obtained in $p\bar{p}$ experiments). It is obvious that a state with the quantum numbers $J^{PC} = 1^{+-}$ cannot be produced in e^+e^- collisions. The decay $\psi' \to h_{c1} + \gamma$ is forbidden by the conservation law of C parity. At a τcF, by observing the decay

$$\psi' \to h_{c1} + \pi^0$$

(in which the isotopic spin is changed), the $^1P_1$ state of charmonium can be discovered with relative ease.[20] At the same time, the spin and parity of the h_{c1} state can be determined.

The charmonium state 2^1S_0 (η_c') has been observed in only one experiment.[21] In this experiment, the branching ratio of the decay $\eta' \to \eta_c + \gamma$ was found to be

$$Br(\psi' \to \eta_c' + \gamma) = (0.75 \pm 0.55)\%.$$

At a τcF, there will be the possibility for detailed study of the state η_c'. Study of decay channels such as $\eta_c' \to \varphi\varphi$ will make it possible to determine the spin and parity of this state.[20]

The width of the decay $\eta_c \to 2\gamma$, calculated in the nonrelativistic quark model, is related to the width of the decay $J/\psi \to e^+e^-$ as follows:

$$\Gamma(\eta_c \to 2\gamma) = 3e_c^2(1 + 1.96\alpha_s/\pi)\Gamma(J/\psi \to e^+e^-) \cong 7 \text{ keV}. \quad (9)$$

Note that the derivation of (9) includes the assumption that at the origin the wave functions of the states η_c and J/ψ are the same. If allowance is made for relativistic corrections, the contribution of the gluon condensate, the spin dependence of the quark interaction potential, and other effects, then

$$\Gamma(\eta_c \to 2\gamma) = 3 - 15 \text{ keV}.$$

The mean of all the existing data is

$$\Gamma(\eta_c \to 2\gamma) = 8.0 \pm 2.2 \text{ keV}.$$

It is clear that an accurate measurement of the $\eta \to 2\gamma$ decay width would be exceptionally important from the point of view of understanding charmonium dynamics. Such measurements could be made at a τcF.[22]

Another important process that could be investigated at a τcF is the purely electromagnetic decay $J/\psi \to 3\gamma$. The width of this decay is given by

$$\Gamma(J/\psi \to 3\gamma) = \tfrac{4}{3}\pi\alpha e_c^4(\pi^2 - 9)\Gamma(J/\psi \to e^+e^-). \quad (10)$$

From this relation, we obtain for the branching ratio

$$Br(J/\psi \to 3\gamma) \cong 2\cdot 10^{-5}.$$

It follows from the existing experimental data that

$$Br(J/\psi \to 3\gamma) < 5.5\cdot 10^{-5}.$$

At a τcF the $J/\psi \to 3\gamma$ probability could be measured by detecting "tagged" J/ψ events:[22]

$$\psi' \to \pi^+\pi^- + J/\psi \quad \llcorner\!\!\to 3\gamma.$$

In such a case, problems do not arise with the background from the process $e^+e^- \to 3\gamma$.

In the decay

$$J/\psi \to \gamma + X$$

gluonium states with mass in the interval 1.5–2 GeV can be produced (lattice calculations, in particular, indicate the existence of gluonium states with masses in this interval). There are two gluonium candidates: $\eta(1440)$ and $f_2(1720)$. Detailed study at a τcF of the various decay channels of these particles and determination of their spins and parities will make it possible to test whether or not $\eta(1440)$ and $f_2(1720)$ are gluonium states.[23]

We conclude with the two following remarks, which are related to recent LEP experiments to measure the number of neutrino species.

1. In recent LEP experiments, the following values were obtained for the "number of neutrino species":[2]

$n_\nu = 2.90 \pm 0.12$ ALEPH;

$n_\nu = 2.93 \pm 0.09$ DELPHI;

$n_\nu = 3.01 \pm 0.14$ L3; $n_\nu = 3.0 \pm 0.4 \pm 0.2$ OPAL.

As is well known, the value of n_γ is determined from the "invisible" decay width of the Z boson (this width is the difference between the total width and the total hadronic and leptonic widths). It is obvious that the LEP data rule out the existence of neutrinos of a fourth species. However, these data do not rule out decay of the Z boson into heavy neutral particles (supersymmetric particles, heavy neutral bosons,...). It would be interesting to study decays of virtual Z particles in which the undetected decay products have different energies. In the studies of Ref. 24, such decays of the ground states of quarkonium $[J/\psi, \gamma, (TT)_0]$ were considered. At a τcF, it will in principle be possible to study the decay

$$\psi' \to J/\psi + \pi\pi, \quad J/\psi \to \nu\bar{\nu}$$

in which the J/ψ particles are "tagged" by measuring the pion momenta.[25] Using the standard model, we obtain

$$R_c = \frac{\Gamma(J/\psi \to \nu_l \bar{\nu}_l)}{\Gamma}$$

$$= \frac{1}{32} \frac{M_\psi^4}{(M_Z^2 - M_\psi^2)^2} \frac{v_c^2}{\sin^4\theta_W \cos^4\theta_W}$$

$$\times \left[e_c^2 + \frac{g_V e_c v_c}{2\sin^2\theta_W \cos^2\theta_W} \frac{M_\varphi^2}{M_Z^2 - M_\psi^2} + \frac{(g_V^2 + g_A^2) v_c^2}{16 \sin^4\theta_W \cos^4\theta_W} \frac{M_\psi^4}{(M_Z^2 - M_\psi^2)^2} \right]^{-1}. \tag{11}$$

Here, $\Gamma(J/\psi \to \nu\nu)$ is the width of the decay of the J/ψ particle into a neutrino–antineutrino pair ($l = e, \mu, \tau$); M_ψ is the J/ψ mass; M_Z is the mass of the Z boson; and g_V, g_A, and v_c are the constants that occur in the lepton and quark neutral currents, respectively, Bearing in mind that $\sin^2\theta = 0.230$ and $M_z = 91.10$ GeV, we obtain $R = 1.1 \times 10^{-7}$ and $\mathrm{Br}(J/\psi \to \nu\bar{\nu}) = 2.3 \times 10^{-8}$. It is expected that the number of ψ particles produced at a tau–charm factory in a year will be about 6×10^9 (at the τcF planned at Dubna the expected number of ψ' particles will be an order of magnitude greater). Bearing in mind that $\mathrm{Br}(\psi' \to J/\psi\pi\pi) \sim 0.5$, we conclude that at a "standard" tau–charm factory about 70 decays $J/\psi \to \nu\bar{\nu}$ will be observed in a year.

2. We consider the results of experiments to measure the number of neutrino species from the point of view of the hypothesis of neutrino mixing. As is well known, the most general neutrino mixing scheme is the one that corresponds to the so-called Dirac and Majorana mass term.[26] In this scheme

$$\nu_{lL} = \sum_{i=1}^{n+m} U_{li} \chi_{iL}. \tag{12}$$

Here, ν_{lL} is the current field of the neutrinos, χ_i is the field of Majorana neutrinos with mass m_i, $1 < m < n$ (the index l takes n values $e, \mu, \ldots$). Thus, in the case of Dirac and Majorana mixing of n current fields the ν_{lL} are superpositions of the left components of $n + m$ fields of Majorana neutrinos with definite masses. Using the standard expression for the neutral neutrino current

$$j_a^Z = \sum_{l=e\ldots} \bar{\nu}_{lL} \gamma_a \nu_{lL},$$

and also taking into account the relation (12), we obtain

$$n_\nu = 3 - \sum |a_{ik}|^2 (1 - R_{ik}). \tag{13}$$

Here $a_{ik} = \Sigma_l\, u_{li}^* u_{lk}$, and $R_{ik} = \Gamma(Z \to \nu_i \nu_k)/\Gamma_0 \leqslant 1$ is a known function of the masses of the particles (a phase factor), and Γ_0 is the width of decay of the Z boson into a neutrino–antineutrino pair, calculated in the standard model. Note that in (13) we have used the condition of unitarity of the mixing matrix U, and also (in accordance with the experimental data) we have assumed that in nature there exist three charged leptons (three neutrino species). It follows from (13) that in the case of neutrino mixing the experimentally measured n_ν can be less than three.[27] We assume that there exist three light neutrinos with $m_i \ll M_Z$ and a heavy Majorana particle with mass $m_4 > M_Z$. Note that such an assumption corresponds to the so-called rocking mechanism,[28] which is the most popular mechanism of neutrino mass generation. For n_γ, we obtain in this case[29]

$$n_\nu = 3 - 2a_{44} + a_{44}^2, \tag{14}$$

where $a_{44} = \Sigma_{l=e,\mu,\tau} |u_{4l}|^2$. We can find $|u_{l4}|^2$ from analysis of data on μ decay and other weak processes. Using the values obtained in this manner, we have[29]

$$(n_\nu)_{mix} = 2.92 \pm 0.19,$$

which does not contradict the LEP data on the measurement of n. It is also clear that an increase in the accuracy of the measurement of n is of exceptional interest for neutrino physics. If future experiments should give indications that $n < 3$, then mixing of light neutrinos with heavy Majorana particles would be one of the possible explanations of such a result. However, there are other mechanisms that lead to $n < 3$. In conclusion, we give a relationship between observable quantities that follows from the neutrino mixing model:

$$n_\nu = 2\frac{B_\tau}{R_\tau^0} + 2\frac{R_\pi}{R_\pi^0} - 1.$$

Here

$$R_\tau = \frac{\Gamma(\tau^+ \to e^+ \nu_e \bar{\nu}_\tau)}{\Gamma(\mu^+ \to e^+ \nu_e \nu_\mu)}, \quad R_\pi = \frac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)},$$

and $R_{\tau,\pi}^0$ are the corresponding ratios calculated in the standard model. As can be seen from this relation, an accurate measurement of the $\tau \to e\nu_e\nu_\tau$ decay width at a τcF could be important for testing the hypothesis of mixing of light neutrinos with heavy Majorana particles.

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